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Suppression of the Spiral Wave and Turbulence in the Excitability-Modulated Media

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Abstract Periodical forcing is used to control the spiral wave and turbulence in the modified Fithzhugh-Nagumo equation (MFHNe) when excitability is changed. The decisive parameter ε of (MFHNe), which describes the ratio of time scales of the fast activator u and the slow inhibitor variable v, is supposed to increase linearly to simulate the excitability modulation in the media. In the numerical simulation, a local periodical stimulus is imposed on the left border of the media and the periods of external forcing are adjusted according to the approximate formula $\omega \propto 1/\varepsilon^{1/3}$ so that using the most appropriate frequency for the external forcing can approach a shorter transient period. It is found that the spiral wave and turbulence can be removed successfully by using an appropriate periodical forcing on the left border of the media. The mean activator and distribution of frequency of all the sites are also used to analyze the transition of spiral wave.

Keywords Spiral wave · Turbulence · Parameter shift and excitability

1 Introduction

Spiral wave patterns are often observed in the excitable media [1, 2], and the spiral wave can step into turbulence resulting from the Eckhaus instability, Doppler instability or meandering instability [3–7]. In the last decades, most studies focused on the spiral wave in the chemical and biological system. For example, many fundamental investigations of the spiral wave have been carried out extensively with the Belousov-Zhabotinsky (BZ) reaction [8–11] and the cardiac tissue [12–17]. The periodical forcing [18–23], feedback [24–27] and excitability modulation [28, 29] schemes are proposed to prevent the appearance and instability of the spiral wave. It is argued that the external currents forcing on the cardiac

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tissue causes a parametric resonant drift [17] and thus the excitability is changed, the action of the external current on cardiac tissue can be described with the parameter shift action. In real surface reaction, the controllable physical parameter set ($P_{\mathrm{CO}}, P_{\mathrm{O}_2}, T$ are pressures of CO and oxygen and the crystal temperature) is mapped into the model parameter (a, b, ε) [30]. The characteristic parameter in excitable media is parameter ε which describes the ratio of time scales of the fast activator u and the slow inhibitor variable v. Heat production is an important problem when higher periodical electric current and/or illumination are used to suppress the useless spiral wave and spiral turbulence in the excitable media. The excitability is changed when the heat production begins to accumulate greatly. Therefore, it is reasonable to simulate the heat production effect on excitability as parameter is shifted. It is found that heat production can enhance the excitability in the media though it is not clear to illustrate the deterministic relation between the parameter shift and the heat production accumulation. In this paper, we introduce the shift of parameter ε to simulate the excitability modulation and external periodical forcing with appropriate period is imposed on the left border of the media according to the approximate formula [32] for calculating the rotating frequency of the spiral wave. Our aim is to remove the spiral wave and spiral turbulence with external periodical forcing in the MFHNe when the parameter shift is in consideration.

2 Parameter Shift in the Reaction-Diffusion System

The modified Fithzhugh-Nagumo equation [31] with two-variables is described as

$$\begin{cases} \dfrac{\partial u}{\partial t} = \varepsilon^{-1} u(1-u)\left(u - \dfrac{v+b}{a}\right) + D\nabla^2 u \\ \dfrac{\partial v}{\partial t} = f(u) - v \end{cases} \tag{1}$$

where u and v describe the activator and inhibitor variables respectively, D is the diffusion coefficient, and the nonlinear function $f(u)$ satisfies $f(u) = 0$ at $u < 1/3$, $f(u) = 1 - 6.75u(u-1)^2$ at $1/3 \le u \le 1$ and $f(u) = 1$ at $u > 1$. The decisive parameters are b (positive for the excitable medium) and ε (the ratio of time scales of the fast activator u and the slow inhibitor variable v). The system owns one stable point $(u = 0, v = 0)$ and two additional unstable points. Suitable initial conditions and parameters are chosen to generate a stable rotating spiral wave with $a = 0.84$, $b = 0.07$, $\varepsilon = 0.004$ and $D = 1$. In our numerical simulation tests, the distance between the adjacent sites $\Delta x = \Delta y = 100/256$ and the time step $h = 0.02$. The Euler forward difference algorithm has been applied for the simulation with the non-flux boundary conditions being used.

The controlled MFHNe with parameter shift is described by

$$\begin{cases} \dfrac{\partial u}{\partial t} = \varepsilon(t)^{-1} u(1-u)\left(u - \dfrac{v+b}{a}\right) + D\nabla^2 u + I\cos(\omega t)\delta_{\alpha i}\delta_{\beta j} \\ \dfrac{\partial v}{\partial t} = f(u) - v \end{cases} \tag{2}$$

where I is the intensity of the external forcing, ω is the angle frequency, α, β, i and j are integers, $\delta_{\alpha i} = 1$ at $\alpha = i$; $\delta_{\alpha i} = 0$ at $\alpha \ne i$; $\delta_{\beta j} = 1$ at $\beta = j$; $\delta_{\beta j} = 0$ at $\beta \ne j$. Clearly, the external forcing is added to the right side of the first formula in (1), it can be regarded as a pure forcing on a square area in the media. For example, the perturbation $k\cos(\omega t)\delta_{\alpha i}\delta_{\beta j}$

is understood as the external electric current. In the case of BZ reaction, $k\cos(\omega t)\delta_{\alpha i}\delta_{\beta j}$ is regarded as the external illumination.

In the following, the ratio parameter ε is supposed to be changed due to some uncertain perturbation and it may be reasonable to describe the excitability change with the shift on ration parameter ε

$$\varepsilon(t) = \begin{cases} \varepsilon_0, & t \le t_0 \\ \varepsilon_0 + k(t - t_0), & t_0 \le t \le t_0 + \Delta t = t_{01} \\ \varepsilon_0 + k(t_{01} - t_0) = \varepsilon_{01}, & t \ge t_0 + \Delta t \end{cases} \tag{3}$$

ε_0 is the ratio parameter without external forcing, the parameter ratio is supposed to increase linearly after a appropriate transient period. Heat production can be accumulated resulting from some external stimulus. It requires a short transient period to diffuse the heat production induced by the external forcing (illumination or electric shock), then the excitability begins to decrease slowly, and it reaches a threshold because the excitability cannot be changed infinitely.

3 Numerical Simulation and Discussions

In the numerical simulation, $\varepsilon_0 = 0.04$, $t_0 = 100$ time units, $\Delta\varepsilon = 0.024$ and 0.032 are used according to the intensity of external forcing. In this section, the external periodical forcing is imposed on a local square area with the size about 5×5 sites, and then the periodical forcing is introduced into a longer but narrower area in the left border in the media. We will check this scheme and observe the competition between the new target wave and the spiral wave and spiral turbulence.

According to the numerical results from Fig. 1, the snapshots of the activator confirmed that the spiral wave is removed by the new target wave as the local periodical forcing begins to work on the left border of the media. The curve for the evolution of the mean activator

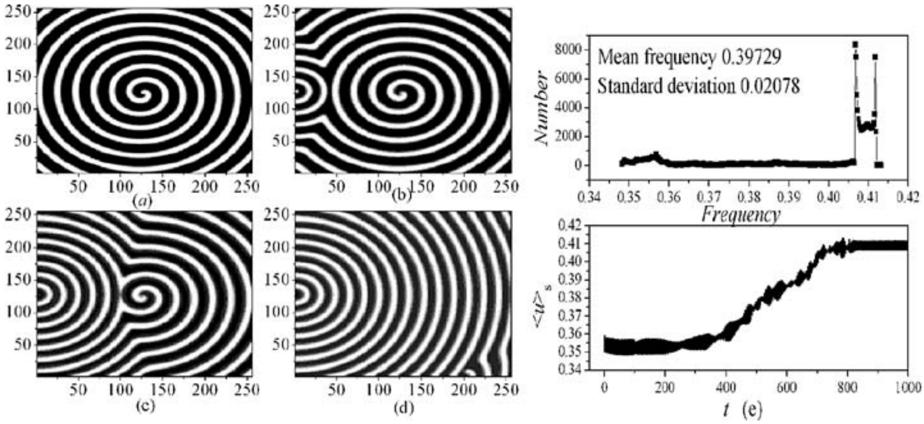

Fig. 1 Evolution of the spiral wave, $\Delta t = 400$, $\Delta\varepsilon = 0.024$, $\varepsilon(t) = 0.04 + 0.6 \times 10^{-4}(t - 100)$ at $100 \le t \le 500$, intensity of periodical forcing $I = 5$, $\omega = 1.6$, for $t = 100$ (**a**), 400 (**b**), 600 (**c**), 800 (**d**), time units, the control area for $\alpha = 1, 2, 3, 4, 5$ and $\beta = 126, 127, 128, 129, 130$, the snapshots are shown in *gray*. The distribution of frequency of all the sites and the evolution of mean activator vs. time are shown in (**e**)

of all the sites shows that the mean activator begin to vary regularly as a stable target wave overcomes the stable rotating spiral wave. The two peaks in the curve of the distribution of frequency for the mean activator of all sites confirms that clear changes are induced as the periodical forcing and parameter shift is introduced into the whole media. Furthermore, it is found that it takes about 600 time units to remove the spiral wave by using the new target wave for $\omega = 1.7$ at $I = 5$, while it takes about 1800 time units for $\omega = 1.5$ and 2400 time units for $\omega = 1.4$ at $I = 5$. To our surprise, the new target wave tried to travel toward the right side in order to drive the tip of the spiral wave out the media, but the new target wave was inundated by the spiral turbulence resulting from the instability of the spiral wave in the right side for $\omega = 1.8, 1.9$ etc. Extensive numerical simulation results show that smaller angle frequency for $\omega < 1.4$ can not induce a new target wave at all. Clearly, the effective angle frequency should be about $1.4 \le \omega \le 1.7$, and the most appropriate angle frequency for the external periodical forcing is $\omega = 1.7$. In this case, $\Delta\varepsilon = 0.024$, the ratio parameter ε can vary from $\varepsilon = 0.04$ to $\varepsilon = 0.064$ so that the parameter requirement can be satisfied for stable rotating spiral wave and meandering spiral wave. In our numerical simulation results, it is observed that the spiral wave can be suppressed successfully as the local periodical forcing begins to work on the left border of the media by using appropriate angle frequency.

Now we investigate the problem when the ratio parameter ε changes from $\varepsilon = 0.04$ to $\varepsilon = 0.072$ for $\Delta\varepsilon = 0.032$, and the competition between the target wave and spiral turbulence is investigated.

The snapshots of Fig. 2 confirm that local periodical forcing-induced target wave can still remove the spiral turbulence and the target will occupy the whole media finally. The curve for the evolution of mean activator of all the sites shows that the mean activator begins to change regularly, corresponding to a stable target wave, as the periodical forcing with appropriate intensity and frequency is introduced into the whole media. The two peaks of the curve for the distribution of frequency of mean activator of all sites indicate the transition of frequency as the local periodical forcing and parameter shift are considered. Furthermore, it takes longer transient period to suppress the spiral wave and turbulence when smaller frequency ω is used with $I = 6$, the numerical results confirm that it will take about 1600 time

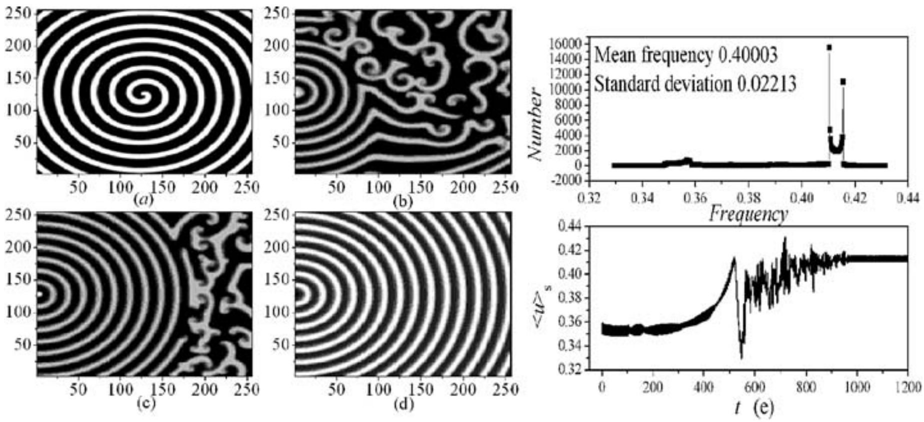

Fig. 2 Evolution of the spiral wave, $\Delta t = 400$, $\Delta\varepsilon = 0.032$, $\varepsilon(t) = 0.04 + 0.8 \times 10^{-4}(t - 100)$ at $100 \le t \le 500$, intensity of periodical forcing $I = 6$, $\omega = 1.5$, for $t = 100$ (**a**), 600 (**b**), 800 (**c**), 1000 (**d**), time units, the control area for $\alpha = 1, 2, 3, 4, 5$ and $\beta = 126, 127, 128, 129, 130$, the snapshots are shown in *gray*. The distribution of frequency of all the sites and the evolution of mean activator vs. time are shown in (**e**)

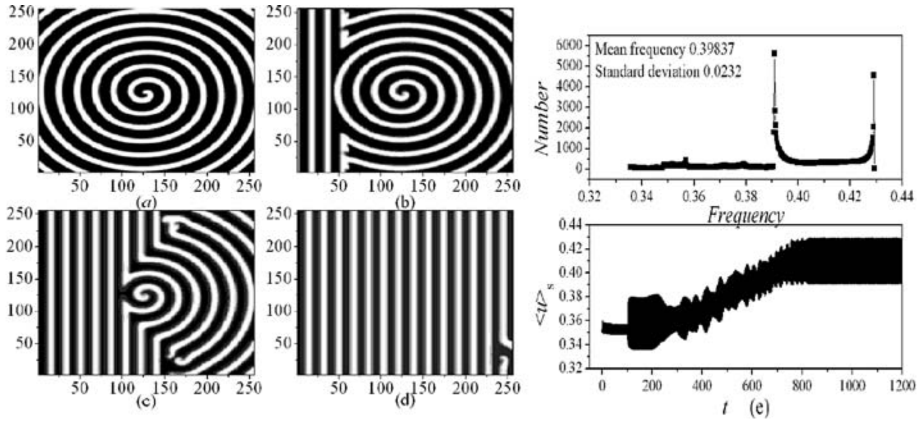

Fig. 3 Evolution of the spiral wave, $\Delta t = 400$, $\Delta\varepsilon = 0.024$, $\varepsilon(t) = 0.04 + 0.6 \times 10^{-4}(t - 100)$ at $100 \le t \le 500$, intensity of periodical forcing $I = 5$, $\omega = 1.6$, for $t = 100$ (**a**), 400 (**b**), 600 (**c**), 800 (**d**) time units, the control area for $\alpha = 1, 2, 3$ and $\beta = 0$, the snapshots are shown in *gray*. The distribution of frequency of all the sites and the evolution of mean activator vs. time are shown in (**e**)

units for $\omega = 1.45$ and about 1800 time units for $\omega = 1.4$. Then extensive numerical studies are given to investigate the cases for $\omega = 1.6, 1.7, 1.8$ and 1.2, it is found that no evident appearance of target wave and the whole media is occupied with the spiral turbulence. Extensive numerical studies show that appropriate angle frequency ω equals 1.3–1.5. No target wave can be generated if the angle frequency $\omega > 1.5$ or $\omega < 1.3$ into this case.

Above all, the local periodical forcing-induced target wave is used to remove the spiral wave and spiral turbulence and the parameter shift is in consideration. In the case of a stable rotating single-arm spiral wave, the tip of the spiral wave often lies in the center of the media, clearly, it is more effective to impose the periodical forcing on the tip directly, and a shorter transient period will cost to remove the useless spiral wave and/or spiral turbulence in the media. It is also interesting to investigate the effect when the periodical forcing is imposed on a long and narrow area in the left border of the media instead of a local square in the media.

The snapshots in Fig. 3 show that the stable spiral wave can be eliminated by the new traveling wave. The curve for mean activator of all sites illustrates that the mean activator begins to change regularly as the new developed traveling wave occupies the whole media and the two peaks in the curve for distribution of indicates the transition of frequency and competition of the stable spiral wave and traveling wave.

The snapshots in Fig. 4 find that the spiral turbulence is induced with increasing ratio parameter ε and the spiral turbulence is removed by the new generated traveling wave as the periodical forcing is imposed on the left border of the media. It is also found in the curve of mean activator of all sites that the mean activator begins to change regularly as the new traveling wave occupies the whole media. Furthermore, the two peaks in the curve of distribution of frequency indicate the change and transition of frequency as the periodical forcing and the parameter shift are considered altogether.

We have investigated the suppression of spiral wave and turbulence in the excitable and the excitability shift is also considered. It is important to discuss principle of excitability shift, which is described with the ratio parameter shift. It seems unimaginable to suppose the parameter shift described with (3). Up to our knowledge, any external forcing or stimulus can induce fluctuation for the activator and inhibitor in the excitable media, thus the ratio

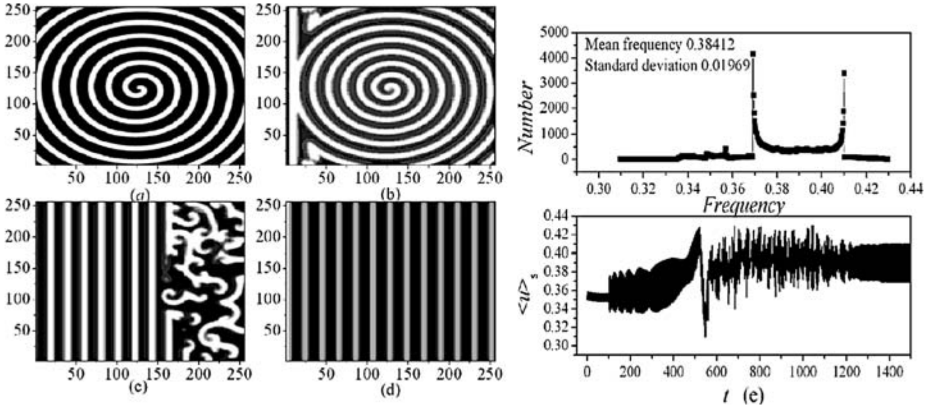

Fig. 4 Evolution of the spiral wave, $\Delta t = 400$, $\Delta\varepsilon = 0.032$, $\varepsilon(t) = 0.04 + 0.8 \times 10^{-4}(t - 100)$ at $100 \le t \le 500$, intensity of periodical forcing $I = 6$, $\omega = 1.5$, for $t = 100$ (**a**), 500 (**b**), 1000 (**c**), 1800 (**d**) time units, the control area for $\alpha = 1, 2, 3$ and $\beta = 0$, the snapshots are shown in gray. The distribution of frequency of all the sites and the evolution of mean activator vs. time are shown in (**e**)

of the activator and the inhibitor is changed. In fact, the rotating period and frequency of the spiral wave is changed as long as any external forcing is imposed on the media. As we know, the rotating frequency of the spiral wave is determined by the external controllable parameter in the media or physical parameter in the model. As result, parameter shift is inevitable as the external forcing begins to work on the media. But the most important thing is what the crucial parameter is? What is the most sensitive parameter in the excitable media? In the case of the MFHNe, the ratio parameter ε can be the most sensitive parameter in the excitable media because any external can change the activator and the inhibitor directly, thus the excitability is changed greatly. D. Barkley et al. suggested that an approximate formula [32] can be used to estimate the rotating frequency of the spiral wave in the Barkley and FHN model, the approximate formula requires $\omega \propto 1/\varepsilon^{1/3}$, it is found that any shift for the parameter ε will induce sharp change for the rotating frequency of the spiral wave. As we known, the effective and powerful frequency of the external periodical forcing is the frequency that equals to the intrinsic frequency of the spiral wave. Therefore, any external forcing on the media will induce the frequency shift of the rotating spiral wave. We have not ascertained how the excitability is changed by the heat production accumulation, but it may be reasonable to suppose that it costs a transient period to change the excitability of the media and the excitability will be enhanced linearly and slowly vs time and finally reach a stable state.

4 Conclusions

In this paper, the external periodical forcing is proposed to eliminate the useless spiral wave and spiral turbulence in the excitable media when the parameter shift is varied linearly. Compared to the previous works about periodical forcing in the excitable media, it shows some difference and advantages. The main results can be listed as the following. (I) The excitability change is considered when the external periodical forcing is used to remove the useless spiral wave and spiral turbulence in the excitable media. (II) It supposes that the excitability of the media is changed linearly and slowly after an appropriate transient period to diffuse the accumulative heat production in the media before it reaches a stable state. (III) The most powerful and effective frequency of the external periodical forcing should be adjusted as the excitability is changed synchronically. (IV) It still confirms that the local periodical forcing is useful and powerful to remove the useless spiral wave and spiral turbulence in the excitable media even if the parameter shift (which caused by the heat production or other uncertain factors) is in consideration. It also implies that the frequency of the external periodical forcing should be adjusted synchronously according to the approximate formula $\omega \propto 1/\varepsilon^{1/3}$. It is more interesting to investigate the practical problem in experiments. For example, how long does it take to response to the so-called heat production effect on excitability? What is the critical intensity and duration of the external periodical forcing to induce visible shift for excitability? We hope that these interesting questions can be further discussed and investigated in the future works.

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References

1. Kapral, R., Showalter, K. (eds.): Chemical Waves and Patterns. Kluwer Academic, Dordrecht (1995)
2. Winfree, A.T.: When Time Breaks Down. Princeton University Press, Princeton (1987)
3. Bär, M., Brusch, L.: Breakup of spiral waves caused by radial dynamics: Eckhaus and finite wavenumber instabilities. New J. Phys. **6**, 5 (2004)
4. Fenton, F.H., Cherry, E.M., Hastings, H.M.: Multiple mechanisms of spiral wave breakup in a model of cardiac electrical activity. Chaos **12**, 852 (2002)
5. Mitkov, I., Aranson, I., Kessler, D.: Meandering instability of a spiral interface in the free boundary limit. Phys. Rev. E **54**, 6065 (1996)
6. Zhou, L.Q., Ouyang, Q.: Experimental studies on long-wavelength instability and spiral breakup in a reaction-diffusion system. Phys. Rev. Lett. **85**, 1650 (2000)
7. Sandstede, B., Scheel, A.: Absolute versus convective instability of spiral waves. Phys. Rev. E **62**, 7708 (2000)
8. Zaikin, A.N., Zhabotinsky, A.M.: Concentration wave propagation in two-dimensional liquid-phase self-oscillating system. Nature (Lond.) **225**, 535 (1970)
9. Marts, B., Martinez, K., Lin, A.L.: Front dynamics in an oscillatory bistable Belousov-Zhabotinsky chemical reaction. Phys. Rev. E **70**, 056223 (2004)
10. Tung, C.K., Chan, C.K.: Dynamics of spiral waves under phase feedback control in a Belousov-Zhabotinsky reaction. Phys. Rev. Lett. **89**, 248302 (2002)
11. Liao, H.M., Zhou, L.Q., Zhang, C.X., et al.: Wave grouping of a meandering spiral induced by Doppler effects and oscillatory dispersion. Phys. Rev. Lett. **95**, 238301 (2005)
12. Shajahan, T.K., Sinha, S., Pandit, R.: Spiral-wave dynamics depend sensitively on inhomogeneities in mathematical models of ventricular tissue. Phys. Rev. E **75**, 011929 (2007)
13. Bernus, O., Verschelde, H., Panfilov, A.V.: Spiral wave stability in cardiac tissue with biphasic restitution. Phys. Rev. E **68**, 021917 (2003)
14. Qu, Z.L., Weiss, J.N., Garfinkel, A.: From local to global spatiotemporal chaos in a cardiac tissue model. Phys. Rev. E **61**, 727 (2000)
15. Xie, F.G., Qu, Z.L., Weiss, J.N., Garfinkel, A.: Interactions between stable spiral waves with different frequencies in cardiac tissue. Phys. Rev. E **59**, 2203 (1999)
16. Roth, B.J.: Frequency locking of meandering spiral waves in cardiac tissue. Phys. Rev. E **57**, R3735 (1998)
17. Panfilov, A.V., Müller, S.C., Zykov, V.S.: Elimination of spiral waves in cardiac tissue by multiple electrical shocks. Phys. Rev. E **61**, 4644 (2000)
18. Schrader, A., Braune, M., Engel, H.: Dynamics of spiral waves in excitable media subjected to external periodic forcing. Phys. Rev. E **52**, 98 (1995)
19. Zhang, H., Hu, B.B., Hu, G.: Suppression of spiral waves and spatiotemporal chaos by generating target waves in excitable media. Phys. Rev. E **68**, 026134 (2003)

20. Zykov, V.S., Bordiougov, G., Brandtstädter, H., et al.: Periodic forcing and feedback control of nonlinear lumped oscillators and meandering spiral waves. Phys. Rev. E **68**, 016214 (2003)
21. Sakaguchi, H., Fujimoto, T.: Elimination of spiral chaos by periodic force for the Aliev-Panfilov model. Phys. Rev. E **67**, 067202 (2003)
22. Gray, R.A., Mornev, O.A., Jalife, J., et al.: Control of traveling waves in the mammalian cortex. Phys. Rev. Lett. **87**, 168104 (2001)
23. Mantel, R.M., Barkley, D.: Periodic forcing of spiral waves in excitable media. Phys. Rev. E **54**, 4791 (1996)
24. Zykov, V.S., Bordiougov, G., Brandtstädter, H., et al.: Global control of spiral wave dynamics in an excitable domain of circular and elliptical shape. Phys. Rev. Lett. **92**, 018304 (2004)
25. Hu, G., Xiao, J.H., Chua, L.O., et al.: Controlling spiral waves in a model of two-dimensional arrays of Chua's circuits. Phys. Rev. Lett. **80**, 1884 (1998)
26. Zykov, V.S., Mikhailov, A.S., Müller, S.C.: Controlling spiral waves in confined geometries by global feedback. Phys. Rev. Lett. **78**, 3398 (1997)
27. Grill, S., Zykov, V.S., Müller, S.C.: Feedback-controlled dynamics, of meandering spiral waves. Phys. Rev. Lett. **75**, 3368 (1995)
28. Karma, A., Zykov, V.S.: Structure of the resonance attractor for spiral waves in excitable media. Phys. Rev. Lett. **83**, 2453 (1999)
29. Kantrasiri, S., Jirakanjana, P., Kheowan, O.-U.: Dynamics of rigidly rotating spirals under periodic modulation of excitability. Chem. Phys. Lett. **416**(4–6), 364 (2005)
30. Bär, M., Gottschalk, N., Eiswirth, M., Ertl, G.: Spiral waves in a surface reaction: model. Calculations. J. Chem. Phys. **100**, 1202 (1994)
31. Hildebrand, M., Bär, M., Eiswirth, M.: Statistics of topological defects and spatiotemporal chaos in a reaction-diffusion system. Phys. Rev. Lett. **75**, 1503 (1995)
32. Margerit, D., Barkley, D.: Selection of twisted scroll waves in three-dimensional excitable media. Phys. Rev. Lett. **86**, 175 (2001)